Completing the square, using the positive sign of the radical,

$$r(a^2-b^2)(b^2\cos^2\theta-a^2\sin^2\theta)=ab(a^2+b^2)\sqrt{(b^2\cos^2\theta+a^2\sin^2\theta)}\text{.........}(6).$$

Multiplying both sides of (6) by $r(b^2\cos^2\theta-a^2\sin^2\theta)$, squaring, and putting in the values from (1) and (2),

$$(a^2-b^2)^2(b^2x^2-a^2y^2)^2-a^2b^2(a^2+b^2)^2(b^2x^2+a^2y^2)=0\text{.........}(7).$$

This is one factor of the given expression. Using the negative sign after completing the square in (3), and employing (4) and (5),

$$(a^2-b^2)\ (b^2\cos^2\theta-a^2\sin^2\theta)r\sqrt{(b^2\cos^2\theta+a^2\sin^2\theta)}\ [r\sqrt{(b^2\cos^2\theta+a^2\sin^2\theta)}+ab]=0\text{........}(8).$$

Equating the last factor to zero, rationalizing, using (1) and (2), we have $a^2y^2+b^2x^2-a^2b^2$ as a second factor.

Also solved by G. B. M. Zerr.

251. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\dfrac{1}{n+1}+\dfrac{1}{2(n+2)}+\dfrac{1}{3(n+3)}+$ etc., $=$

$$\frac{1}{n^2}+\tfrac{1}{2}\left[\frac{1}{n-1}+\frac{1}{2(n-2)}+\frac{1}{3(n-3)}+\text{.....}+\frac{1}{l(n-l)}\right],$$

l being equal to $n-1$, n being any positive integer greater than one.

Solution by L. E. NEWCOMB, Los Gatos, Cal.

The general term is, $\dfrac{1}{r(n+r)}=\dfrac{1}{nr}-\dfrac{1}{nr(r+n)}$. Let $r=1, 2, 3,$ .... in succession; then $\dfrac{1}{n+1}=\dfrac{1}{n}-\dfrac{1}{n(n+1)}$, $\dfrac{1}{2(n+2)}=\dfrac{1}{2n}-\dfrac{1}{n(n+2)}$, $\dfrac{1}{3(n+3)}=\dfrac{1}{3n}-\dfrac{1}{n(n+3)}$.

$\therefore$ Sum $=\dfrac{1}{n}-\dfrac{1}{n(n+1)}+\dfrac{1}{2n}-\dfrac{1}{n(n+2)}+\dfrac{1}{3n}-\dfrac{1}{n(n+3)}$.......... and all the terms after the rth vanish.

$$\therefore \text{Sum}=\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}+\text{.....}+\frac{1}{n^2}\equiv\frac{1}{n^2}+\left[\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}+\text{.....}+\frac{1}{ln}\right]\ (1).$$

In the series (2) $\dfrac{1}{n-1}+\dfrac{1}{2(n-2)}+\dfrac{1}{3(n-3)}+\text{.....}+\dfrac{1}{l(n-l)}$, the general

term is $\frac{1}{r(n-r)}=\frac{1}{nr}+\frac{1}{n(n-r)}$, and since $\frac{1}{n-1}=\frac{1}{n}+\frac{1}{n(n-1)}$, $\frac{1}{2(n-2)}=\frac{1}{2n}+\frac{1}{n(n-2)}$, $\frac{1}{l(n-l)}=\frac{1}{ln}+\frac{1}{n(n-l)}\equiv\frac{1}{n(n-1)}+\frac{1}{n}$, the sum of (2)= $\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}\text{..............}\frac{1}{n}+[\frac{1}{n(n-1)}+\frac{1}{n(n-2)}+\text{.....}+\frac{1}{n}]$. The terms within and without the parenthesis are now plainly identical; consequently, $\frac{1}{2}[\frac{1}{n-1}+\frac{1}{2(n-2)}+\frac{1}{3(n-3)}+\text{.....}+\frac{1}{l(n-l)}]$ substituted for $\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}$ .... $\frac{1}{ln}$ in the right hand member of (1) will satisfy the equation.

II. Solution by R. D. CARMICHAEL, Hartselle, Alabama.

Represent the series of the first member by S_n. Then,

$$S_2=\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+\frac{1}{4.6}\text{.....}=\tfrac{1}{2}[(1-\tfrac{1}{3})+(\tfrac{1}{2}-\tfrac{1}{4})+(\tfrac{1}{3}-\tfrac{1}{5})+(\tfrac{1}{4}-\tfrac{1}{6})\text{.....}]=\tfrac{1}{2}(1+\tfrac{1}{2}).$$

$$S_3=\frac{1}{1.4}+\frac{1}{2.5}+\frac{1}{3.6}+\text{.....}=\tfrac{1}{3}[(1-\tfrac{1}{4})+(\tfrac{1}{2}-\tfrac{1}{5})+(\tfrac{1}{3}-\tfrac{1}{6})+(\tfrac{1}{4}-\tfrac{1}{7})\text{.....}]=\tfrac{1}{3}(1+\tfrac{1}{2}+\tfrac{1}{3}).$$

....

$$S_n=\frac{1}{n}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}\text{.....}+\frac{1}{n})=\frac{1}{n^2}+\frac{1}{n}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}\text{.....}+\frac{1}{n-1})$$

$$=\frac{1}{n^2}+\frac{1}{n}(\frac{1}{n-1}+\frac{1}{n-2}+\frac{1}{n-3}\text{.....}+\tfrac{1}{3}+\tfrac{1}{2}+1)$$

$$\therefore 2S_n=\frac{2}{n^2}+\frac{1}{n}[(1+\frac{1}{n-1})+(\tfrac{1}{2}+\frac{1}{n-2})+(\tfrac{1}{3}+\frac{1}{n-3})\text{.....}+(1+\frac{1}{n-1})].$$

$$\therefore S_n=\frac{1}{n^2}+\tfrac{1}{2}[\frac{1}{n-1}+\frac{1}{2(n-2)}+\frac{1}{3(n-3)}+\text{..........}\frac{1}{l(n-l)}],\ l\text{ being equal to }n-1.$$

Also solved by G. W. Greenwood, Henry Heaton, and G. B. M. Zerr.

252. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x-y=\frac{1}{3}\pi$; (2) $\sin x=\cos^3 y$.

Solution by J. SCHEFFER, Hagerstown, Md.

Since $x=y+\frac{1}{3}\pi$, we have $\sin x=\frac{1}{2}\sin y+\frac{1}{2}\sqrt{3}\cos y$.

$\therefore\frac{1}{2}\sin y+\frac{1}{2}\sqrt{3}\cos y=\cos^3 y$.

$\therefore 1-\cos^2 y=4\cos^6 y-4\sqrt{3}\cos^4 y+3\cos^2 y$, or $4\cos^6 y-4\sqrt{3}\cos^4 y+4\cos^2 y-1=0$; putting $\cos^2 y=z$, we get $z^3-\sqrt{3}z^2+z-\frac{1}{4}=0$; putting $z=t+\frac{1}{3}\sqrt{3}$, we get $t^3=\frac{1}{4}-\frac{1}{9}\sqrt{3}$; $\therefore t=(\frac{1}{4}-\frac{1}{9}\sqrt{3})^{\frac{1}{3}}$.